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FORSCHUNGSARBEIT

„Quantum Mechanics in Phase Space“

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Quantum Mechanics in Phase Space

Bálint Koczor and Frederik vom Ende are PhD students in the International Doctoral Programme “Exploring Quantum Matter” (ExQM), which brings together TU-Munich, LMU-Munich, and the Max-Planck-Institute for Quantum Optics (MPQ). Their respective background is in chemical physics and in mathematics.

This recent collaboration [arXiv:1811:05872] introduced parity operators in a rich family of quantum phase-space distributions previously obtained via convolutions, integral transformations, or Fourier transforms. Our generalised parity operator allows for direct calculations as exemplified for various phase spaces incl. the Born-Jordan distribution. We also obtained a generalised spectral decomposition of the BJ-parity operator, proved its boundedness, and gave its matrix representation in the number-state basis.

In this representation, the approach naturally connects to quantum optics, where it is of conceptual importance.

Quantum Mechanics in Phase Space GPCRs

Phase spaces as given by the Wigner distribution function provide a natural description of infinite-dimensional quantum systems. They are an important tool in quantum optics and have been widely applied in the context of time-frequency analysis and pseudo-differential operators. Phase-space distribution functions are usually specified via integral transformations or convolutions which can be averted and subsumed by using (displaced) parity operators proposed in this work.

Parity Operators

Building on earlier work for Wigner distribution functions [A. Grossmann, Comm. Math. Phys. 48, 191 (1976)], *parity operators* give rise to a general class of distribution functions in the form of quantum-mechanical expectation values. We relate these distribution functions to the so-called Cohen class [L. Cohen, J. Math. Phys. 7, 781 (1966)] and recover various well-known quantization schemes and distribution functions from the literature now in a more general frame.

Born-Jordan Quantisation

The Born-Jordan distribution and its parity operator constitute a most remarkable instance among the phase-space approaches with a distribution function satisfying the marginal conditions and therefore allowing for a probabilistic interpretation. Thus, both the Born-Jordan distribution and its corresponding quantisation scheme have a fundamental importance in quantum mechanics. In particular, there have been several attempts in the literature to find the “proper” quantisation rule for observables using either algebraic or analytical techniques. A recent paper [Found. Phys. **44**, 1096 (2014)] of a senior collaborator in this project, Prof. Maurice A. de Gosson in Vienna, analysed the Heisenberg and Schrödinger pictures of quantum mechanics and showed that for proving the equivalence of both theories, one has to use the Born-Jordan quantisation rule rather than the Weyl rule (which hitherto was common belief).

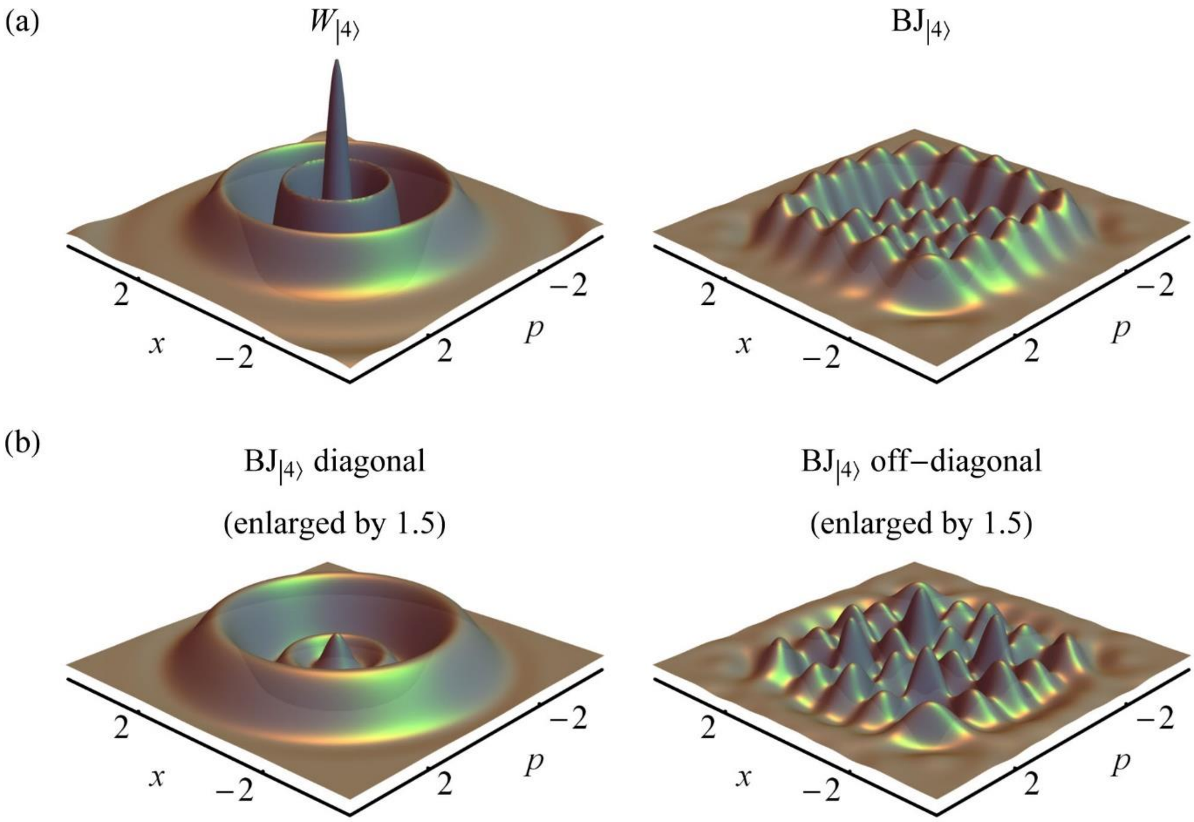

Figure 1 (a) Wigner and Born-Jordan phase-space plots of the number state |4>. (b) The Born-Jordan distribution is decomposed into functions that correspond to diagonal and off-diagonal entries of the parity operator matrix.